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On mapping the world in a mirror

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Abstract. Small, very convex mirrors create images of effectively the whole visual world around themselves. Aspects of the geometry of such images are explored by employing optical ray-tracing techniques in mapping information in the optic array onto a plane. Some applications are suggested for representing and simulating the whole visual field.

Introduction

Reflections in convex mirrors have a fascination all of their own. In a small silvered sphere or hemisphere—perhaps an ornament or utensil—the world is reflected, distorted, and miniature to be sure, but complete. M C Escher perhaps captured the fascination most directly in his well known 1935 lithograph "Hand with Reflecting Globe" (figure 1). John Ashbery's long poem "Self-Portrait in a Convex Mirror" (1976) was inspired by such a mirror in a seventeenth century painting by Francesco Parmigianino. Aldous Huxley might have explained our enchantment as being akin to that we feel for jewels and crystal balls—a concentration of life, an emanation of light.

Now, the observation that the image created by small spheroidal mirrors is of (nearly) the whole environment around the mirror (including oneself) is, of course, quite accurate; that convex mirrors afford wide-angle views is well known. But how might the *geometry* of such images be understood more clearly? How might this understanding prove useful? This short inquiry addresses these questions. The first part outlines an approach to the problem, the central part explores the geometry of what I will call *mirror maps*, that is, these special mappings of the whole visual field, and the last part presents some possible uses and my own experimental work.

The optic array

The concept of the *optic array*, together with the ray-tracing procedures of geometrical optics, provides an interesting approach to the problem of mirror maps. At any point in an environment, light rays from the surrounding surfaces converge from all directions. This set of converging rays—the optic array—is usually unique to the point upon which the rays converge: unique in its distribution of colors and intensities and their changes over time (Gibson, 1966). A great deal of information about the layout and material of the immediate environment thus exists 'in' an optic array at any given point, whether an observer happens to be there or not. Sets of optic arrays—often belonging to a continuous path of movement—contain invariant characteristics which correspond to the permanent, larger physical environment. The eyes of insects, frogs, cats, or people are provided the same optic arrays in the same places: that they pick up different invariants in arrays—see different things in the environment—is a matter of biophysical selectivity and, of course, cognitive processing. The three-dimensional optic array is thus posited as being more fundamental and dynamic than the images that can be derived from it, say, through photography or on a retina. Rather than a convenient logical construction, it is a characteristic of the physical world to which mobile organisms must adapt in order to see.

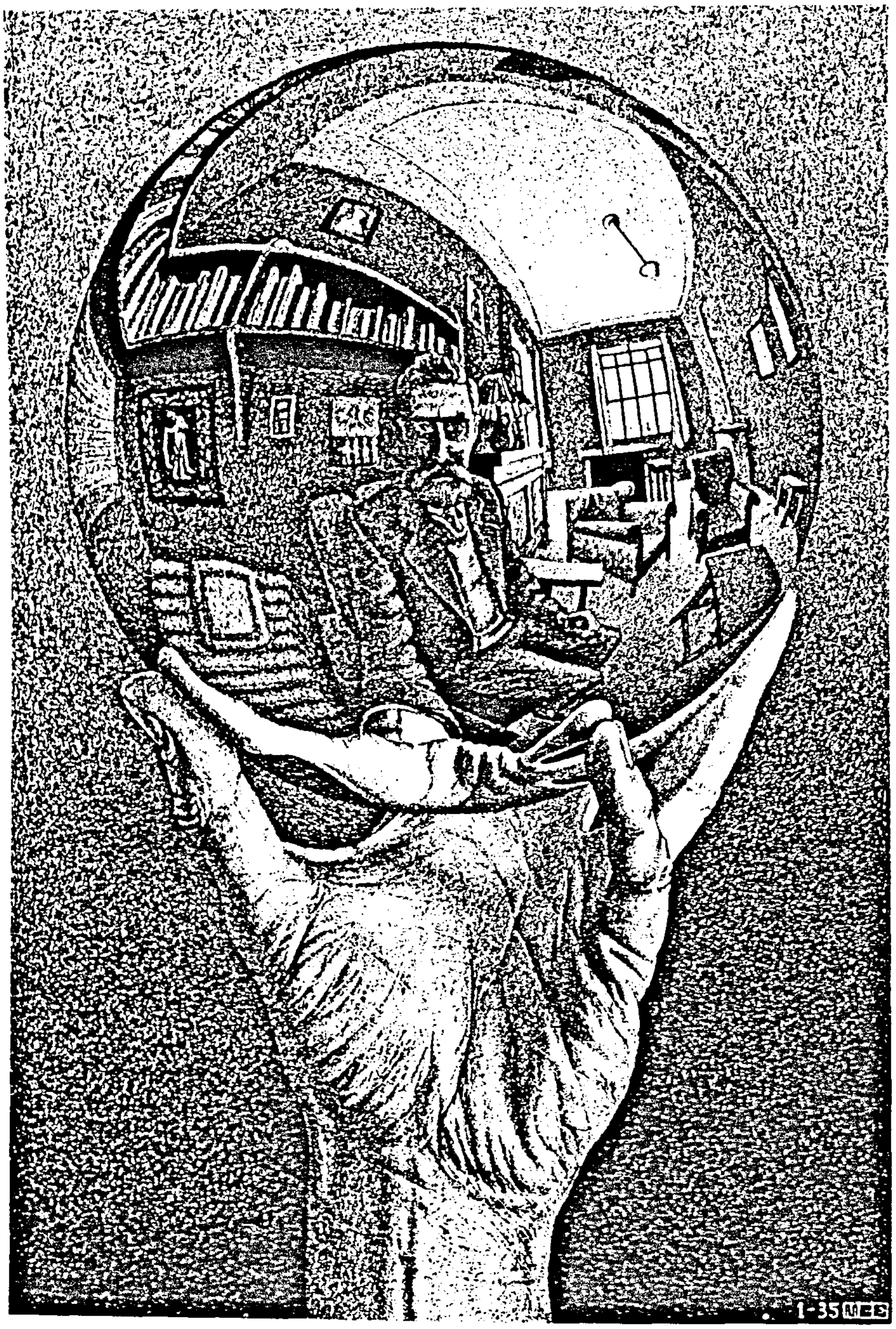


Figure 1. M C Escher's "Hand with Reflecting Globe".

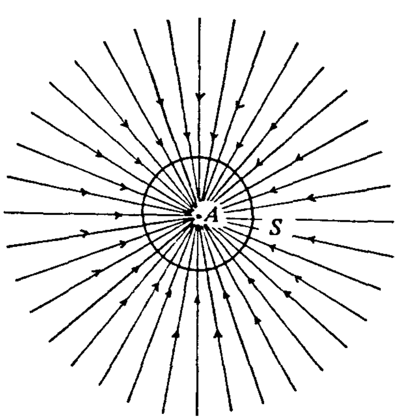


Figure 2. Convergence of light from the environment to a point, A, in the environment.

Most of the information in an optic array at a point A lies in the angular distribution of the color and intensity of light arriving at A, which in turn can be mapped onto any convex, closed surface, S, to which A is internal. Most simply, S is a sphere of an arbitrarily small radius with A at the center (figure 2).

Mirrors

Mirrors may be said to 'edit' optic arrays, cutting, as it were, a section out of the 'natural' optic array at B and splicing into its place a section of another optic array at A [1]. For example, imagine a plane mirror placed midway between A and B, silvered on B's side. The natural part of B's array obscured by the mirror (that is, that part of the array which would 'belong to B' were the mirror not there) is replaced by the natural part of A's array obscured from A by the mirror (figure 3).

When the mirror is plane, the solid angle of A's natural array inserted at B is equal to the solid angle removed from B's natural array, that is, $\psi_A - \psi_B$. If the mirror is convex, however, then ψ_A can be greater than ψ_B [2]. Some of the information at A—that is, that part of it destined for A but occluded by the mirror—is spatially compressed to fit into a comparatively smaller part of the array at B. This kind of precise editing is accomplishable, however, only by a limited number of geometrically 'well-behaved' mirrors, namely, conic surfaces of revolution whose axes of rotation contain A and B (see figure 4). The curve rotated is described mathematically by the formula $r = a/(1+\epsilon\cos\phi)$, where r is the length of the radius of the mirror surface from A, a is a scale constant, ϵ is the *eccentricity*, and ϕ is the angle subtended by r and the line (axis) between A and B ($r = a/\cos\phi$ for the plane mirror).

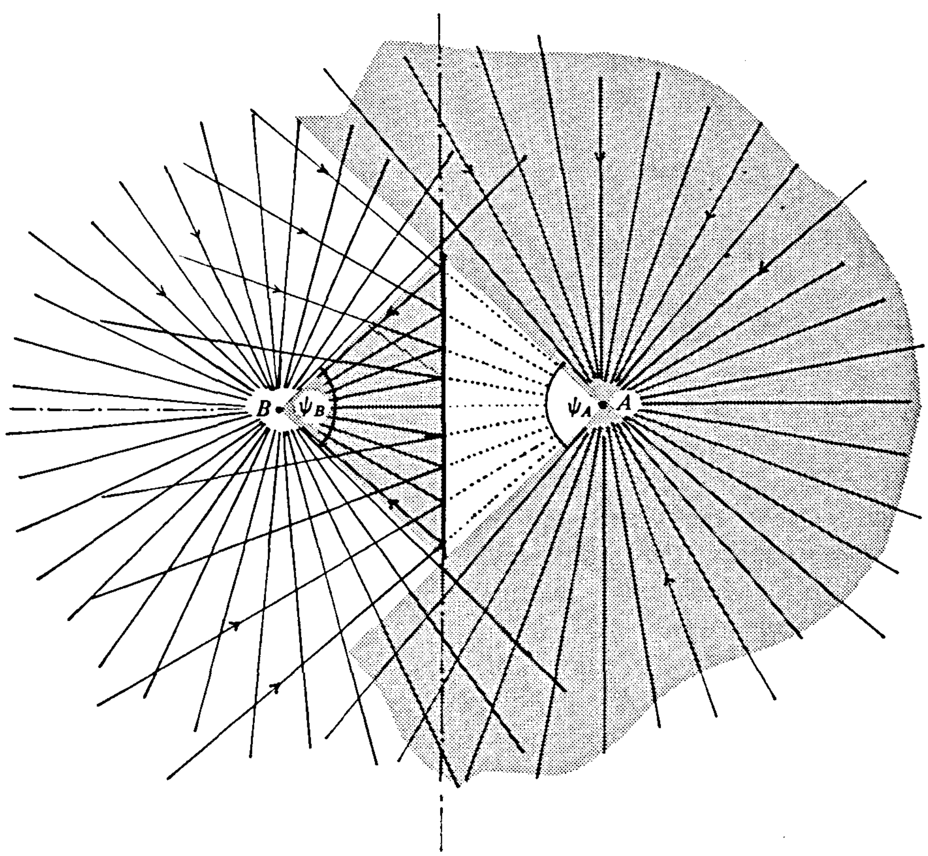


Figure 3. The 'editing' of optic arrays by a mirror.

[1] 'Mirror-reversed', of course.

[2] The inverse is true for concave mirrors.

The aim of this study is to show how convex mirrors can map the optic array at A. Specifically, one would wish to have ψ_A approach 4π (steradians) whereas ψ_B should be somewhat less than 2π. Under these conditions, the full optic array at A can be projected onto a plane of finite size between the mirror and B which in turn can be easily seen, photographed, and drawn.

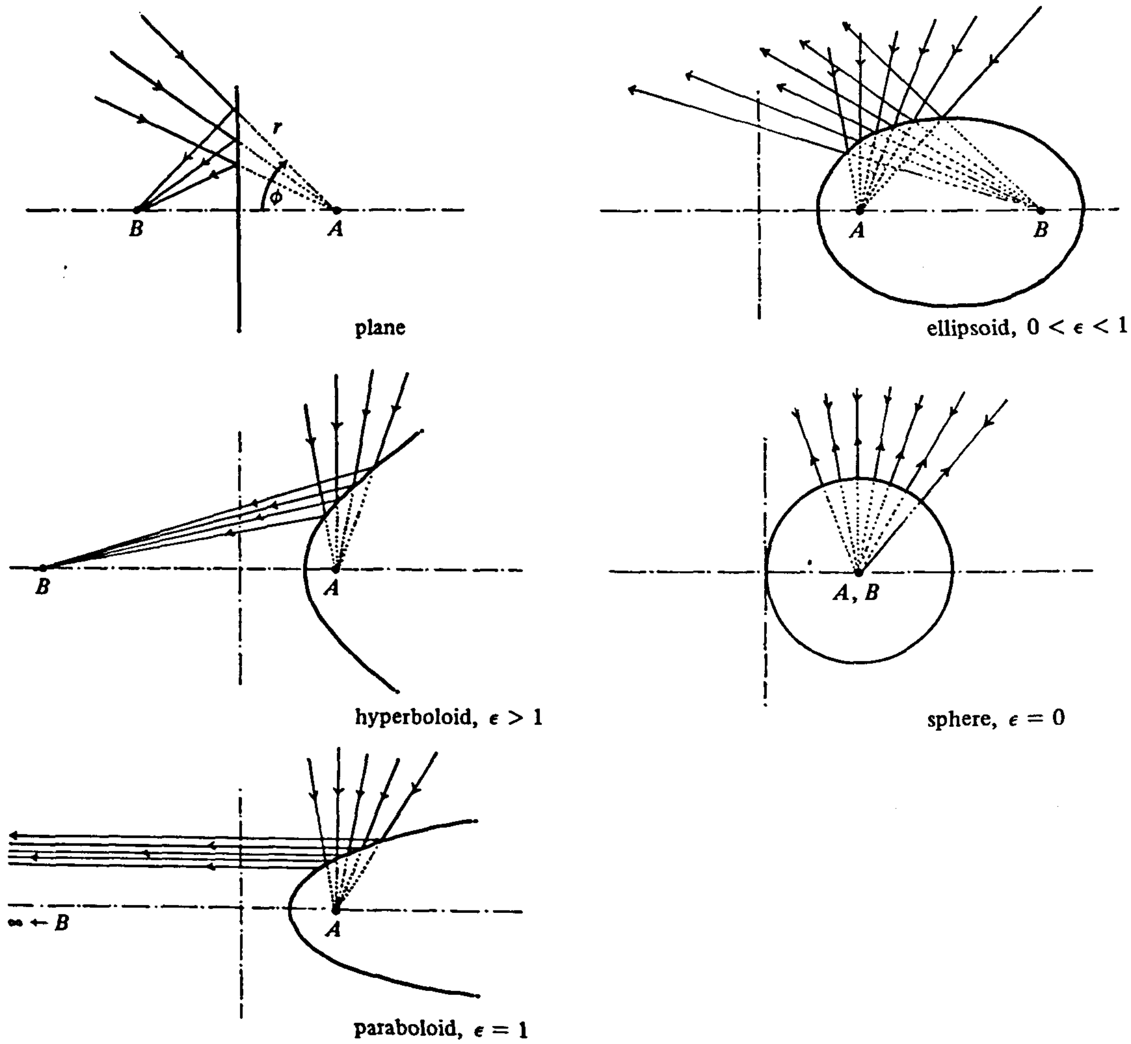


Figure 4. Editing of optic arrays by convex mirrors derived from conic-section surfaces of revolution.

Mirror mapping

From figure 4 one can see that the hyperboloid and paraboloid mirrors are the most promising. In both cases, much of the optic array at A can be projected to an *image plane*, L, placed normal to the A–B axis between the mirror and B. The mirror map thus occurs on the image plane.

A convenient general configuration of mirror, M, image plane or map, L, and points in the environment visible from A is as follows[3]. A is at the origin of a rectangular coordinate system (0, 0, 0), the convex surface of M faces 'up'. L is at some distance above M, normal to the Z-axis.

ϕ is measured from the positive z direction to d, θ from the positive x direction to d. Points in the world thus have coordinates (x, y, z) or (d, ϕ, θ). If b is the width of the mirror, then the map in L is a disc of diameter b in which projected points in the world have map coordinates ρ and θ (figure 5).

(3) For an extended treatment of visible point sets, or *isovists*, see Benedikt (1979) and Davis and Benedikt (1979).

It is fairly straightforward to map (ϕ, θ) into (ρ, θ). (In the next section it is shown how d may be mapped). In the case of the hyperbolic mirror one has:

$$\rho_{\text{hyp}} = \frac{fr\sin\phi}{1 - r\cos\phi}\,, \qquad r = \frac{a\sin\phi}{F(1 + F/2\cos\phi)}\,,$$

where F is the distance between A and B, and f is the distance of L from B (figure 6). The parabola, however, is far simpler (figure 7):

$$\rho_{\text{par}} = r\sin\phi = \frac{a\sin\phi}{1+\cos\phi} = a\tan\frac{\phi}{2}\,.$$

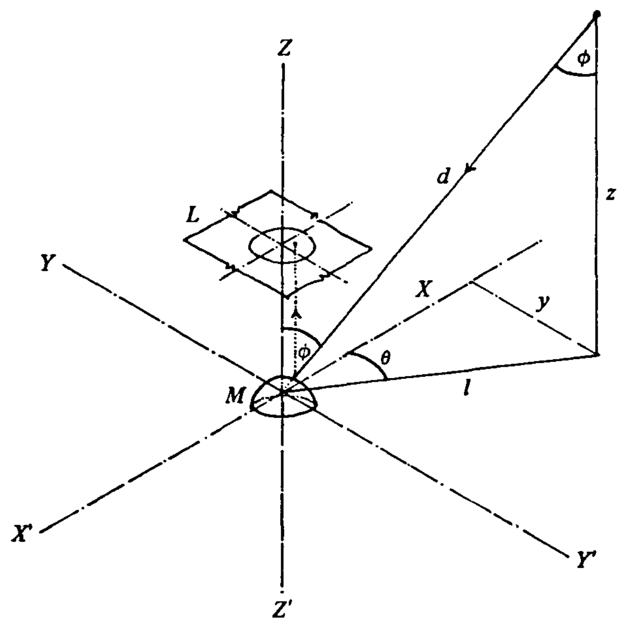
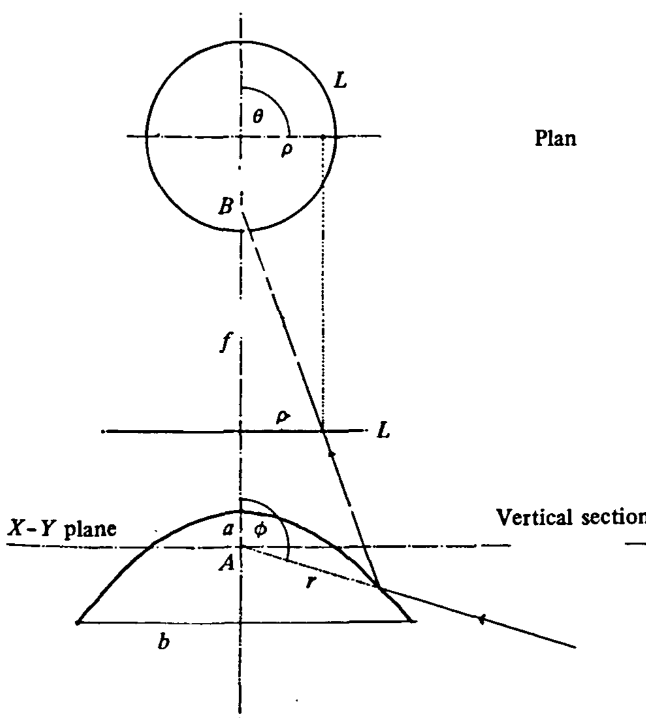


Figure 5. Coordinate system and mirror/map orientation.



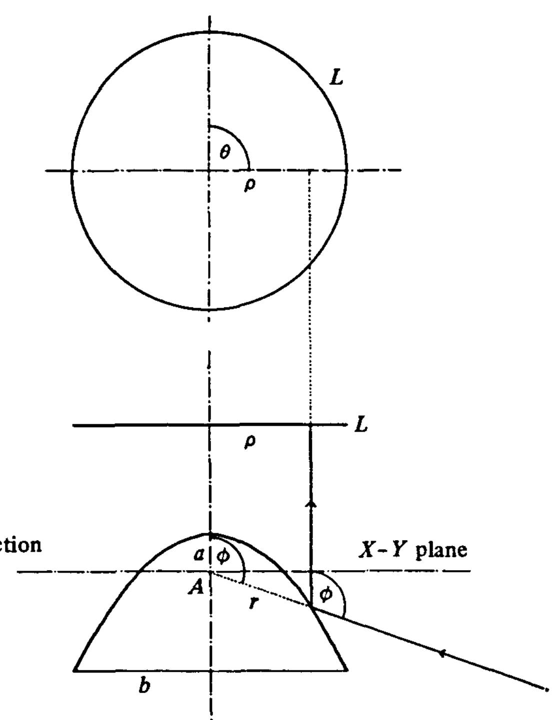


Figure 6. The hyperbolic mirror.

Figure 7. The parabolic mirror.

The hyperbolic and parabolic mirrors both have inherent limits on the upper value which ϕ, and hence ψ_A, can take:

$$\frac{\sin(\phi_{\max})}{1+\epsilon\cos(\phi_{\max})} = \frac{b}{2a}\,, \qquad \text{for the hyperboloid,}$$

$$\phi_{\max} = 2\arctan\left(\frac{2a}{b}\right), \qquad \text{for the parabaloid.}$$

The only finite size mirrors that can have $\phi_{\max} = \pi$ are the ellipsoid and the sphere. But the 'natural' array lines of these mirrors do not lend themselves to the creation of a map in L in quite the same way as was the case for the hyperboloid and paraboloid: those rays that project conveniently from the mirror to the map would not converge at A.

Consider the spherical mirror (which turns out to have the same appealing simplicity as the parabolic mirror). Figure 8 shows a hemispherical mirror of radius a situated with its center at $(0, 0, -\frac{1}{2}a)$. Reflections are mapped by parallel projection onto L. One finds:

$$\phi_{\text{sph}} = \arctan\frac{l}{z} + \arcsin\frac{a\sin(\phi/2)}{d}\,,$$

$$= \arctan\frac{l}{z}\,, \qquad \text{for } d \gg a\,.$$

$$\rho_{\text{sph}} = a\sin\frac{\phi}{2}\,,$$

and

$$g = \tfrac{1}{2}\left(\sec\frac{\phi}{2} - a\right).$$

Here, g is a measure of aberration—indicating where an incident ray from a point in the world that is mapped in L would intercept the Z-axis were the mirror not there.

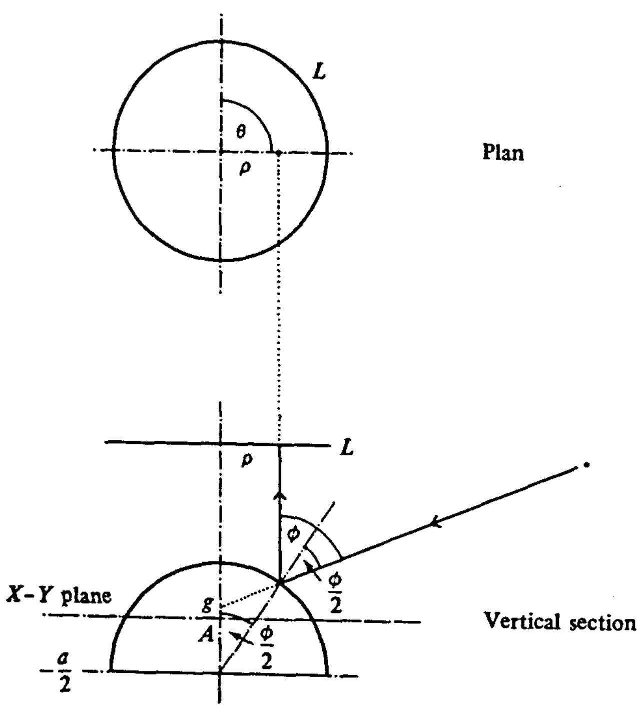


Figure 8. The spherical mirror.

The map in L then, is, strictly speaking, made up of concentric rings corresponding to cones out of adjacent optic arrays that progress upwards from the origin. Note that $g \leqslant \frac{1}{2}a$ for $\phi \leqslant \frac{2}{3}\pi$ [(4)].

Aberration and error in the value of ϕ can be avoided in certain operational contexts, as they are chiefly a consequence of the physical presence and size of the mirror. If a mathematical model is all that is required, it is possible to make a arbitrarily small and to ignore g. Better, one can use a purely geometrical construction to generate the maps. Figure 9 illustrates the construction of (a) the map of the hemispherical mirror and (b) the map of the parabolic mirror. In both, the optic array at A is mapped onto the unit sphere, S, centered on A. These points are then projected from the 'south pole', s, of S: in the case of the spherical mirror to the surface of a hemisphere centered on s, thence by parallel vertical projection to L; in the case of the parabolic mirror, directly to L. The reader may recognize the latter as a stereographic projection of S onto L (figure 10).

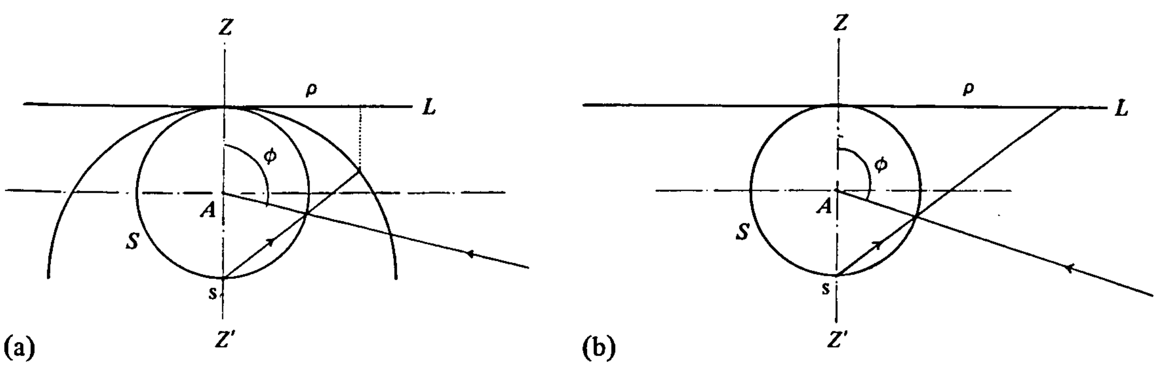
Both in hemispherical and in parabolic mirrors a relationship exists between the solid angle subtended at A by an environmental surface and the area of that surface in the map. If ψ_i is the solid angle of surface i subtended at A and α_i its area in L, then for the hemispherical mirror one finds:

$$\frac{\psi_i}{4\pi} = \frac{\alpha_i}{\pi a^2},$$

that is, $\alpha_i/\psi_i = \frac{1}{4}a^2$ = a constant, for a mirror of a given radius. The parabolic mirror is somewhat less convenient:

$$\frac{\alpha_i}{\psi_i} = \frac{1+\cos\phi_i}{4\pi\rho_i^2},$$

where ϕ_i and ρ_i are measured to the centroid of the surface in world and map respectively.



(a) (b)

Figure 9. Geometrical construction of the image in L: (a) created by the spherical mirror, (b) by the parabolic mirror.

Depth information

Distances from A to points in the environment are not yet mapped. It is necessary to consider relative motion between world and mirror, that is, to pay attention to the 'optical flow' of the array at A as mapped in L (compare also Koenderink and van Doorn, 1975; Lee, 1974).

When there is relative motion, points in L move with a speed and direction that depend in part upon d (the distance to the world-point from A). The velocity vector of such a point in L, $\mathrm{d}R/\mathrm{d}t$, can be analyzed into two components, one in the radial

[(4)] For the hyperbolic and parabolic mirrors $g = 0$, and the image in L is a map of the array at A only.

direction, $\mathrm{d}\rho/\mathrm{d}t$, and one in the circumferential direction, $\mathrm{d}\theta/\mathrm{d}t$, so that

$$\mathrm{d}R^2 = \mathrm{d}\rho^2 + 2\rho(1-\cos\mathrm{d}\theta)(\rho+\mathrm{d}\rho)$$
$$= \mathrm{d}\rho^2 + \rho^2\mathrm{d}\theta^2 .$$

If one rotates the horizontal axes so that, for convenience, the mirror moves (relatively) always along the X-axis, then

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{-\sin\theta}{l}\frac{\mathrm{d}x}{\mathrm{d}t},$$

$$\left(\frac{\mathrm{d}\rho}{\mathrm{d}t}\right)_{\mathrm{sph}} = \frac{-a}{2}\sin\frac{\phi}{2}(1+\cos\phi)\frac{\mathrm{d}z}{\mathrm{d}t},$$

$$\left(\frac{\mathrm{d}\rho}{\mathrm{d}t}\right)_{\mathrm{par}} = \frac{-4a}{l}(1-\cos\phi)\frac{\mathrm{d}z}{\mathrm{d}t},$$

$$\phi_{\mathrm{sph}} = 2\arcsin\frac{\rho}{a}, \quad \text{and} \quad \phi_{\mathrm{par}} = 2\arctan\frac{\rho}{a} .$$

From l one can derive d if required. It will suffice to show then how l is found given only information as to the real mirror velocity vector ($\mathrm{d}x/\mathrm{d}t$, $\mathrm{d}z/\mathrm{d}t$) together with the information in the map θ, ρ, $\mathrm{d}\rho/\mathrm{d}t$, $\mathrm{d}\theta/\mathrm{d}t$ ($\theta \neq 0$):

$$l^2_{\mathrm{sph}} = \frac{-\rho\sin\theta\left(\dfrac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 - 4a(1-\cos\phi)\left(\dfrac{\mathrm{d}z}{\mathrm{d}t}\right)^2}{\left(\dfrac{\mathrm{d}\rho}{\mathrm{d}t}\right)^2 + \left(\rho\dfrac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2},$$

$$l^2_{\mathrm{par}} = \frac{-\rho\sin\theta\left(\dfrac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 - \dfrac{a}{2}\sin\dfrac{\phi}{2}(1+\cos\phi)\left(\dfrac{\mathrm{d}z}{\mathrm{d}t}\right)^2}{\left(\dfrac{\mathrm{d}\rho}{\mathrm{d}t}\right)^2 + \left(\rho\dfrac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2} .$$

The use of mirror maps

The problem of mapping the optic array into a plane is not unlike a classic problem of cartography: making a two-dimensional plane map of the earth's surface. Solutions to this problem vary in their capacity to preserve in the plane a convenient correspondence to the shapes, areas, distances, and directions of surfaces and points on the sphere.

It turns out that the hemispherical and parabolic mirrors both have cartographic counterparts. If one imagines a hemispherical mirror placed at the center of the earth, then the image in L of the earth's surface (or, for that matter, of the sky) turns out to constitute a map known as "Lambert's Azimuthal Equal Area Projection" (Greenhood, 1964, page 151; Kellaway, 1970, page 94), whereas the parabolic mirror, as has been noted, produces a stereographic map in which angular relationships between points in S are preserved in L.

Given that one can map the optic array, or whole visual field, onto a sphere and in turn onto a plane much as cartographers map the globe, of what benefit is the knowledge that these maps can be created by certain mirrors? Most evidently, for the ease with which such maps can be made in real environments. In natural or experimental environments, mirrors such as those discussed can be photographed by using camera lenses of relatively long focal length—superwide-angle photography, if you will. Of course, still photography and cinematography are both possible. Figure 10 is an example of a photographically produced mirror map. Note that the orientation of the mirror can be chosen at will. Figure 11 illustrates the effect of orientation on the organization of the map.

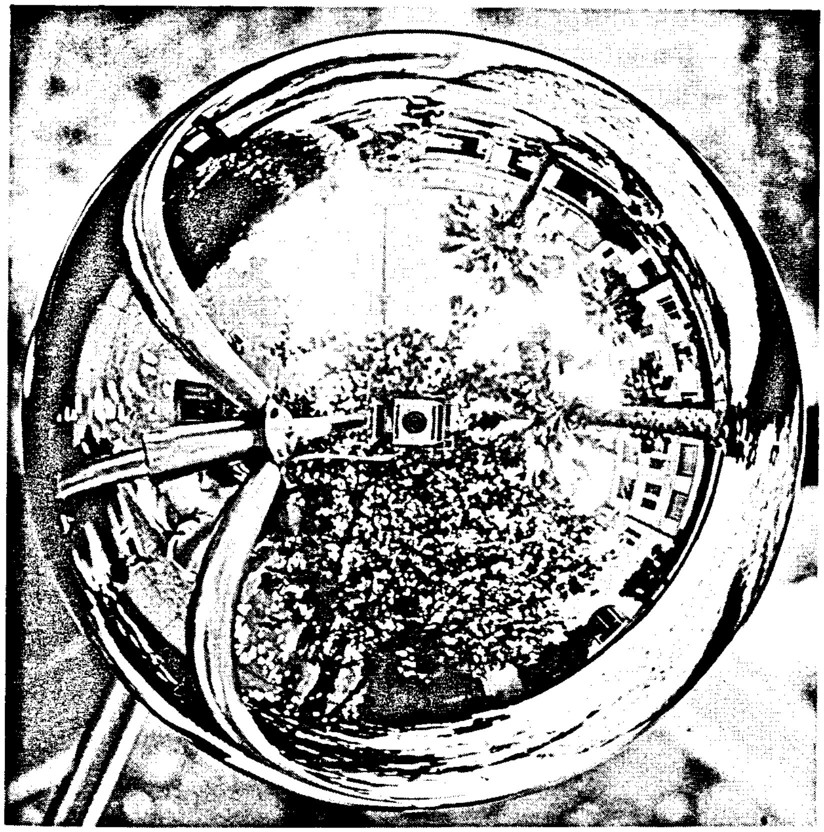


Figure 10. A spherical mirror map, photograph.

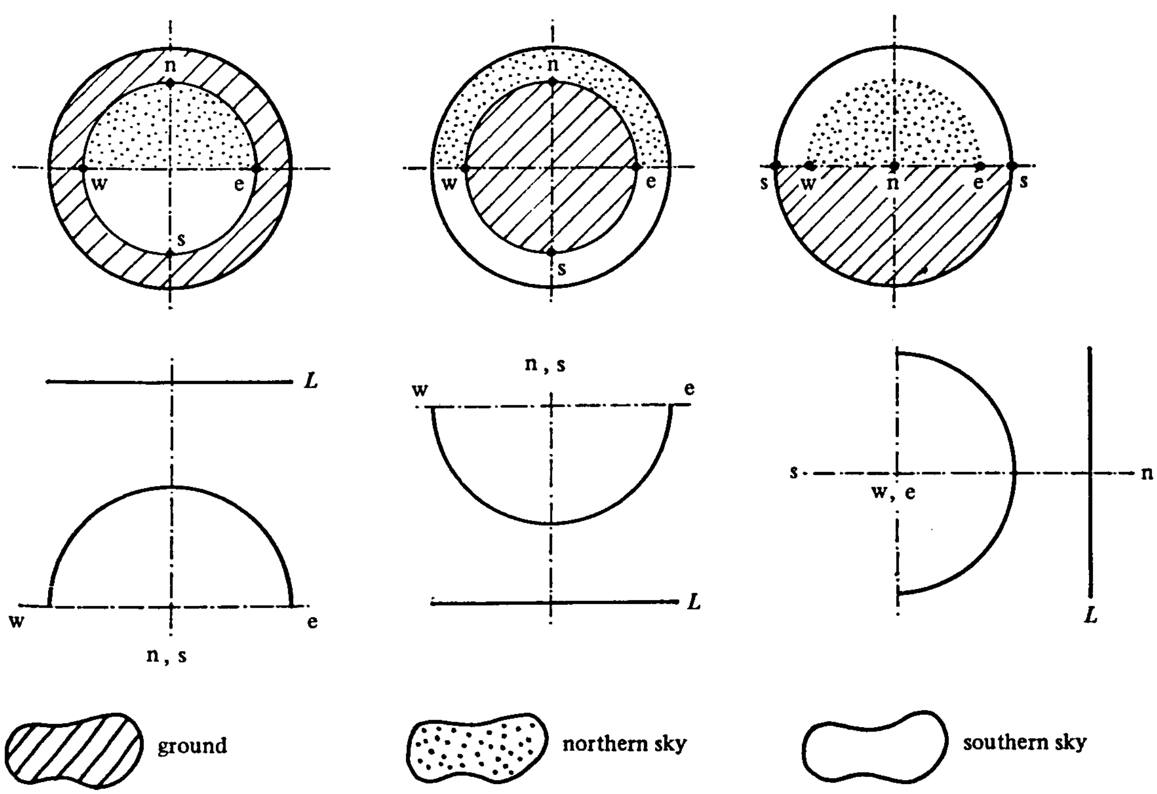


Figure 11. Alternative orientations of a mirror map.

Because the geometry of the world-in-the-map bears a well-understood and systematic relationship to the real world, computer generation of maps is simple. An environment entered and stored as points, lines, and planes in rectangular coordinates is easily transformed into a mirror map at a chosen point of observation. (The 'recognition' of maps—that is, the automatic transformation of certain maps into a general representation of an environment—is, of course, infinitely more difficult.)

Architects have known for some time that the *surrounding* quality of the built (and natural) environment is an essential ingredient in the perception of space and in the appreciation of the 'feel' of a place. Yet adequate means for representing, recording, and simulating the whole visual field, and therefore studying this quality, are lacking. Wide-angle photography (panoramic or fish-eye) or specialized perspective drawing (cylindrical or spherical perspective) often introduces unwelcome and poorly controlled distortions while typically not mapping much more than half the visual field—half an optic array. Psychologists involved in environmental perception or 'human factors' studies are at a similar disadvantage if they wish to investigate, for example, central versus peripheral vision, molar properties and effects of 'optical flow' in space and motion perception, responses to whole visual environments, and so on.

In putting mirror maps to use, however, one must first distinguish whether the task at hand is one of *representation* or *simulation.* In representing the visual field, systematic distortion is acceptable: provided no information is lost, compression and distention in the image can be compensated for or ignored. Hopkinson (1971), for example, was content to calculate view factors from distorted fish-eye photographs. The mirror maps are distorted in just this way.

In simulation, distortion is not acceptable. The geometrical correctness of the (re)created visual field of the observer is essential. Simulations may, however, be made from representations, as and if the latter's distortions are optically corrected. In the cinemascope process, for example, the map—the picture on film—is horizontally compressed; during projection, anamorphic lenses compensate for and cancel the distortion.

Purely as representations, then, mirror maps may provide the basis for a standard mathematical and computational treatment of the optic array: to date, most studies adopt unique coordinate systems, nomenclature, notations, and formulae for essentially the same facts (for example, Clocksin, 1978; Johansson, 1977; Koenderink and van Doorn, 1976; Gibson et al, 1955; Lee, 1976). Nor is the *whole* optic array ever explicitly considered.

That 'mirror map mathematics' might begin to become a standard model would be a less convincing proposal, however, were it not possible to use mirror maps for simulation. For perhaps the most interesting possibilities lie in simulation, particularly, in experimentally creating and manipulating whole or large parts of actual visual fields. In a process analogous to the cinemascope process, a mirror map—photographed, computer-generated, or hand-drafted—can quite literally be optically projected back onto the 'original' mirror to create an image on the inside of a spherical screen surface. At the center of the spherical screen the original (or desired) optic array is (re)created. Such a system is under development by the author(5). The configuration is not unlike that of a planetarium. Figure 12 shows it in a form appropriate to individual studies in motion perception, space perception, peripheral vision, and environment perception, as well as for other uses such as flight simulation, architecture and landscape analysis and design. Figure 13 shows a one-third scale model in operation.

(5) SPHERE—a projection system for recording and recreating the whole visual field. Technical problems and their solution are outside the intended scope of this paper. The research is supported by the University Research Institute, The University of Texas at Austin.

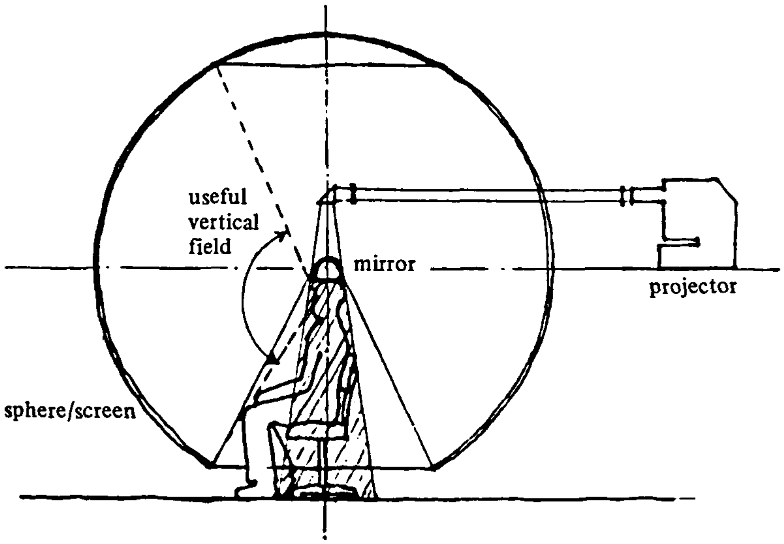


Figure 12. Schematic of the SPHERE spherical projection system.

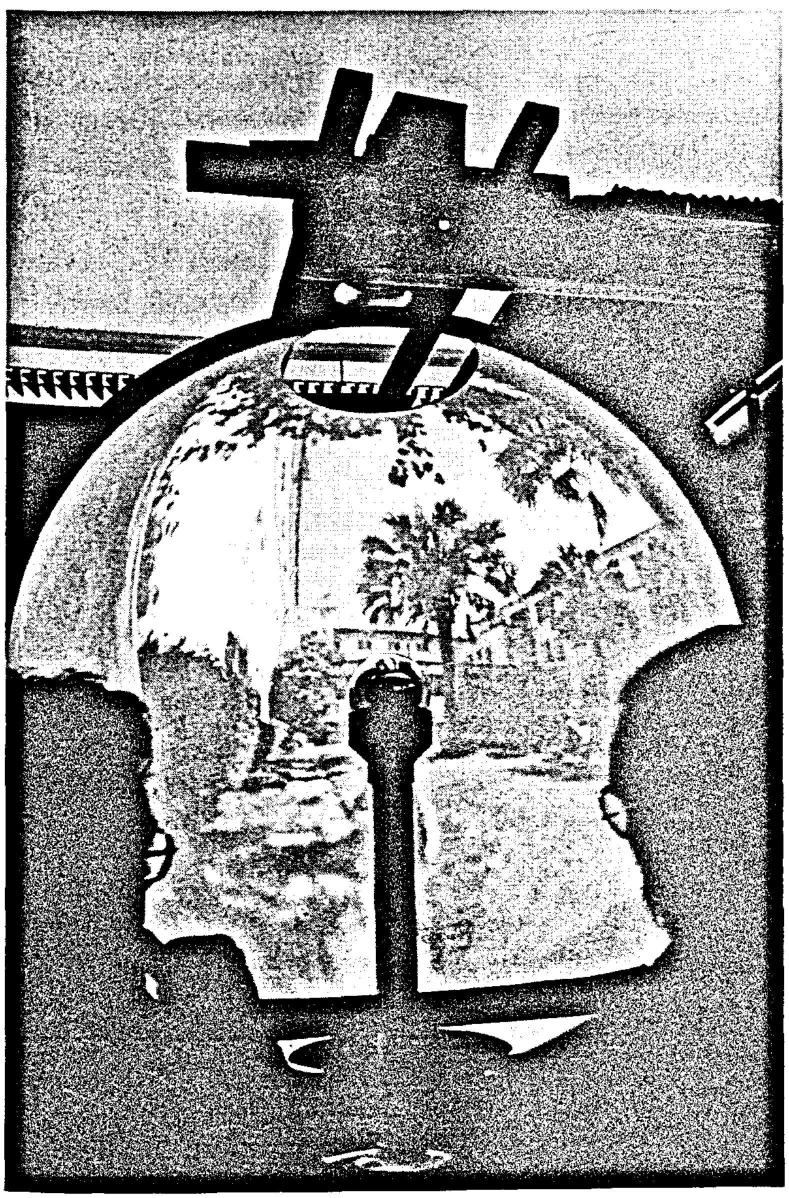


Figure 13. Half of a SPHERE image, working model.

Conclusion

I find it fascinating to consider that a silver sphere placed anywhere in the world would reflect its whole environment to an observer—revealing, as it were, the optic array, the visual information that exists *there* and, by extension, everywhere invisibly until 'seen'. Architectural space, then, need not be thought of as empty or figural, but as a *field* of light-borne information with densities, gradients, sources, and sinks—a view which, to my mind, has interesting implications for architectural theory and theory in environmental psychology (Benedikt, 1977; 1979).

This short paper, however, has attempted only to show how the standard technique of optical ray tracing applied to small spheroidal mirrors—informed and motivated by the notions of 'optic array' and 'information field'—can produce useful mappings of the whole visual field at a point of observation. The geometry and mathematics of mirror maps, introduced here, appears to be concise and manageable. More work in this area should be fruitful, as would further work in the area of simulation along the lines described. Renewed attention by architectural researchers and perception psychologists to the complex, enveloping quality of the visual world is certain to require the kinds of capabilities for representation and simulation that mirror maps provide.

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