A GENERAL THEORY OF VALUE

Appendix Three: **On Omega at Different Scales**

It hardly needs pointing out that reality seems to organize itself—or at least allows *us* to organize it—into spatiotemporal classes of sizes, or "scales." In *space*, big things are made of littler things; little things are made of littler-things-yet...and so on down. In *time*, long-lived movements are made of shorter-lived episodes, episodes of shorter-lived events, these of smaller moments, these of tinier instants...and so on down too. Nomenclature may vary, but the idea stays the same.

Not just material things, but also energy patterns follow this logic. Consider a stretch of open ocean. Upon its surface, at one and the same time, there are tiny ripples made by rain and bubbles, larger waves caused by the wind and by the wakes of boats, and larger and larger ones caused by storms, seismic events, and the tide. In wavelengths varying from millimeters (for bubbles) to thousands of miles (for tidal movements), all of these patterns are superimposed upon each other and "use" the same water. The electromagnetic field too is sortable into a spectrum of wavelengths, from short waves (around 10^{-14} meters for gamma rays) to medium waves (around 10^{-9} meters for visible light) to long waves (around 10^{6} meters for long radio waves). These different patterns in the electromagnetic field coexist distinct from each other and yet superimposed as modulations of the same "stuff."

With informational codes we find the same logic. Language, for example, is made up of sentences, sentences by clauses, clauses by phrases, phrases by words, words by syllables, syllables by phonemes (or letters), phonemes by sonic formants (or letters by ink strokes), and so on down into a microworld of vibrating molecules and atoms and particles fleeter and tinier yet, to scales at which being-part-of-a-language is no longer the dominant organizing principle. Something very similar can be said of computer code, music, machines, living cells, pictures, and DNA. Indeed, what is there that we can leave out?

So obvious is all this that it is easy to pass over the deep mystery of why the world is organized in this way. Our mathematics, our language, our very thought processes, are so saturated with the logic of whole/part, bigger/smaller, inclusion/exclusion, one/many, fleeting/long-lived, that it is difficult to sustain a feeling for the uncanniness of it all or to imagine how it could be otherwise.

I shall not try.

Rather, I will describe one way of representing how different scales in a system can carry different information and perform different functions. More specifically, I will suggest a way of transcribing the results of such an analysis—a "scale analysis"—onto the Ω-surface, an abstraction introduced in Chapter Two. Not everything to be presented in the next few pages finds use in the body of this book. That is why it appears in an appendix. My hope, however, is that the tools fashioned here, and the insights offered along the way, might help future investigators do more with them than I did.

Perhaps the most general representation of a "system"—already a very general idea!—is the *graph*. Not the familiar sort of graph that maps the magnitude of one variable, "x," against the magnitude of another variable, "y," but the sort of graph that has "nodes" and "links" (sometimes called "vertices" and "arcs") that describe a connected set of defined-to-be-atomic parts existing in some spatiotemporal relation to one another and communicating solely along the links indicated. A telephone network as a clear example one such system, with the telephones as the nodes and the wires as links; the interstate highway system between cities might be another. But in truth, the graph is an extremely generalizable and abstractable structure whose logic appears to underlie many phenomena and systems.

A graph is said to be *fully connected* when every node has one link to every other node. If a graph has N nodes and L links, and is fully connected, then $L = L_{max} = N(N-1)/2$.. If it is less-than-fully connected, then L lies somewhere between zero and $N(N-1)/2$. If L is near zero, though, one might hesitate to call the set of nodes a "system" at all. For starting from L_{max} and eliminating links at random, it becomes rapidly less likely, as L passes $2N$ getting smaller, that all the nodes could act together, even indirectly, by relaying information through intermediary nodes. Indeed, perfectly organized, $N-1$ is the smallest number of links that could indirectly join all N *nodes*. To see this, think of a ring of N children holding hands. One child lets go: they're still all in touch. Another lets go, and you either have two groups or someone standing alone. Removing links at random of course, would very likely begin to isolate individual nodes and small groups far sooner.[1]

In the realm of pure mathematics, graphs show connectivity only. In what follows, we will use graphs that map onto real space, i.e. graphs where the relative *lengths* of links are significant too. Consider a city map for example, drawn "to scale," with the front door of every building represented as a node. The set of all links, drawn as straight lines, would represent not only all the possible trips that could be made between buildings, but also all the crow-fly distances between front doors. (Actual driving or walking distances, of course, would slightly different).[2] A structurally similar graph might be constructed for a living cell with nodes representing mitochondria and links between them representing electrochemical pathways, or of an electrical circuit with wires between components like resisters, capacitors, transistors, and the like, where the designers want to minimize the distance between the parts..

We can now begin our analysis.

Figure A3.1 represents a fully connected system of 24 randomly distributed nodes (N = 24) and 276 links ($L = L_{max} = 24(24 - 1)/2 = 276$), broken down into four link-length scales, "short," "medium," "medium-long," and "long." The actual ranges of these scales, and the total number of scales, M, are a matter of choice.[3]

Figure A3.1 Decomposition of a fully connected system of diameter *D* into four subsystems, each using similar-length links, "long", "medium-long," "medium" and "short."

$M = 4$ was the number chosen here for illustrative purposes, and I apportioned the full range of lengths, between zero and the diameter of the system, D, equally between them.[4] To wit, where l is the physical length of a link, the scale named "short" includes only links between 0 and $D/4$ in length (i.e. $0 \leq l_{short} < D/4$); the scale called "medium" includes only links between $D/4$ and $D/2$ in length (i.e. $D/4 \leq l_{med} < D/2$); the scale "medium-long" includes only links between $D/2$ and $3D/4$ in length (i.e. $D/2 \leq l_{med\text{-}long} < 3D/4$); and the scale "long" includes only links between $3D/4$ and D (i.e. $3D/4 \leq l_{long} \leq D$). Adding up the number of links, L, appearing at each scale gives us the total number of links in the graph. That is, $L_{total} = L_{short} + L_{medium} + L_{med\text{-}long} + L_{long}$. Needless to say, no link in the graph can be counted twice, i.e. have two scales.[5] Notice, however, that since the graph fully connected, every *node* is connected to one or more other nodes at all M scales. If it were less-than-fully connected (or had very few nodes) then each node would be connected at somewhere between zero and M scales. All this illustrated in Figure A3.1.[6]

As can be seen from Figure A3.1, the actual number of links appearing at each scale is markedly different. There are far fewer short links than medium-long ones, for example. This is the result of an important, odd, and purely mathematical fact: that the frequency distribution of distances between uniformly (or randomly) distributed points within a circle is not uniform. (Nor, for that matter, is the distribution of distances between points on a line segment, in or on a sphere, or in or on any finite surface or volume, uniform).[7] For a circle, which here serves as our idealized system-cum-graph, the frequency distribution of lengths looks like Figure A3.2. Let us call it the "alpha-distribution."

Figure A3.2 The percentage of all links in a fully connected graph that fall into the four scale categories, "short," "medium," "medium-long" and "long" as defined in Figure A3.1.

The non-uniform nature of the alpha-distribution has some interesting consequences. At random, choose any two points inside a circle. The chances are greatest that the distance between them will be 0.42 of the diameter of the circle. (The actual number is 0.41811.... The rational fraction $\frac{5}{12}$, which is exactly halfway between a $\frac{1}{3}$ and an $\frac{1}{2}$, is close to this number.) *Inside of a circle, there are a greater number of line segments with a length of around 0.42D than there are line segments of any other length.*

At random, choose any two front doors in a city that is roughly circular in shape and that has roughly uniform building density (like Phoenix). The chances are greatest that the distance between those two doors will be around forty two percent of the diameter of the city. It follows that there are a greater number of potential trips of *this* distance than potential trips of any other distance. No real survey of real trip distances in such a city would, of course, produce a smooth curve of the sort produced by the mathematical model of the alpha-distribution. But so strong is the underlying effect that the "hump" at $0.42D$ would make itself felt through the irregularities caused by actual street networks, non-uniform habitation density, zoning, and various topographical features like rivers and mountains.

In the abstract again: take an indefinitely large field of indefinitely many nodes and indefinitely many links connecting them. Establish a finite number of equal-range scales, M. You will not find the alpha-distribution pattern.[8] But carve from this infinite field, anywhere, a disc of diameter D, and instantly the alpha-distribution pops up. The alpha-distribution is the mark of finitude itself: the degree of organization, R, inherent in very act of "carving" or isolating the disc from the limitless larger fabric, plus the degree of organization inherent in the act of sorting it's inter-point distances into a finite number of classes.[9] Take, therefore, any circular subregion of a city (or department store, or...) that happens to have a uniform density of points of interest, and *within that subregion* trips of 0.42 of the subregion diameter will very likely be the most frequent. Do this in reverse: study empirically the statistical distribution of trip lengths in a large urban region. If the distribution does *not* turn out to approximate the alpha-distribution, then it will be found to be the sum of smaller alpha-distributions, the diameter of each of which will be the size of a more-or-less holistic subregion—or "neighborhood." The city will prove to be a loose agglomeration of such neighborhoods. In the case of cities, there are many complicating factors here, to be sure, such as the telephone system's undermining of physical proximity as the basis for community, as well as perduring patterns of specialized land use. But the point of the example is to point to a general tendency in the *structure of space itself* as it were, whenever it is delimited in some way.

Visualize now a circular region with a large number of nodes uniformly or randomly distributed in it as before, but each node working like a radio station, with a transmitter of certain power and a receiver of certain sensitivity. Imagine also that the receivers are of a somewhat peculiar sort: they respond only to signals of a certain strength—too weak and they do not hear the signal, too strong and they are "deafened" by it. Successful communication between any two nodes (we shall assume them all to be identically equipped) thus depends on three factors: (1) the power of their transmitters, (2) the distance between them (because signal strength drops off as the inverse square law dictates), and (3) the sensitivity of their receivers. Let us assume that all nodes are equal in transmitting power but different in receiver sensitivity. It is easy to see that merely by adjusting sensitivity thresholds—upper sensitivity and lower—one or all receivers can be set to hear only signals from transmitters situated at a certain distance or range of distances away: those too close are rejected, those too far are not picked up. Or better, we can have it that *each and every receiver can distinguish and sort signals according to signal strength;* indeed into *M* classes of signal strength. With transmitter strength standardized, we have the basis now for sorting links into *M* scales. Compare Figure A3.3 to Figure A3.1

Figure A3.3. Showing how signal strength combined with receiver sensitivity-thresholds can serve as a scaling device, courtesy of the inverse square law.

The reason for conjuring up the nodes-as-radio-stations image is to point out that the existence of a *link* does not imply an actual cable, conduit, or road running between two nodes as the graphics of graphs and networks might suggest. When nodes can broadcast their signal in every direction as well as pick up the signals of others from every direction, like a radio station or light source, then "links" represent *functional* connections between nodes. (We could just as well have used voices and hearing, or smell and noses, or chemical diffusion within cells as examples.) When all can hear all, the graph representing the system is just as "fully connected" as it would be if a cable joined each to each. And as long as *distance* plays the role it usually does in the physical world (through the inverse square law),[10] a scale analysis becomes possible and the alpha-distribution comes into play.

The nodes-as-radio-stations image also allows us to think about the *cumulative* effect of increasing communication range from zero to D, an idea which can in turn help us understand the quasi-economic constraints that a system faces as it tries in some way to grow or evolve.

What is the information capacity of a system described as a graph with nodes and links? As we learned in Chapter One, this is the same as asking: what level of *complexity* is the system capable of sustaining, C_{pot}? We can predict that C_{pot} will not be evenly distributed among the different scales.

Assume that each link in a system carries one bit of information per unit of time. (What goes on *within* each node we do not know, or, for now, care to know.)[11] Then a fully connected system will have a capacity of $N(N-1)/2$ bits. This is the potential complexity, C_{pot}, of the system as a whole, i.e. at all scales. Now let us look at all M scales. *The alpha-distribution places a maximum on the potential complexity of each scale*, and thus an upper limit too on its actual complexity, C, on its degree of organization, R, and its magnitude of omega, Ω.

Now, fully connected systems are as rare in nature as they are in cultural and mechanical systems. Rarely are all *possible* links between nodes either established or functional. But *all systems operate at different scales simultaneously nonetheless*, with the alpha-distribution creating an invisible ceiling to the magnitude of C_{pot} at various scales. In the abstract, this gives us a picture like Figure A3.4

Figure A3.4. Potential complexity and Omega (hypothetical) at different scales in a single system ($M = 15$)

Now let us imagine that a system is "trying to improve itself," trying, that is, to increase in over-all Ω. Perhaps the system is a firm and its managers are studying communication patterns; or perhaps it is a primitive cell "trying," through trial and error in reproduction, to become more complex-and-organized. Question: at what scale(s) should it concentrate its efforts?

Well, the ones with the greatest inherent capacity for complexity, as we have seen, lie around $0.42D$. Any marked lack in connectivity at other scales is worth attention, *but attention to the scale(s) around 0.42D is likely to be the most rewarded.* If the scales around $0.42D$ are already working at full capacity (i.e. with Ω or C at a maximum) then there are two options: return to look to other, under-utilized scales, or expand physically so that the 0.42D zone shifts to the right in absolute size. Here is one reason for *growth.* So let us talk about "growth" a little more.

Go back to our radio-station example. Setting a receiver's sensitivity so that only transmitters approximately 0.42D units-of-distance away are heard will connect it to the largest number of surrounding transmitters. This we know. Let us now remove this peculiarity of the receivers: let there be no upper limit to the signal strength they can cope with; i.e. let them be "undeafenable." Now what if, bent upon self-improvement, our system realizes that the reason it is so limited is that its transmitter power levels are so weak that only nodes very nearby to each other can communicate? What should it do? The answer is clear: increase transmitter power or increase receiver sensitivity (either will do) so that communication range, g, increases...until it reaches D.[12] *But here is a question: does every incremental increase in communication range reward the system with a concomitant increase in total potential complexity (and thus, indirectly, with the possibility for greater* Ω*)?* The answer is: no. The alpha distribution sees to that, as Figure A3.5 illustrates. Starting from a range of zero, the marginal "profit" per unit increase of range—and we might imagine that it *costs* the system in some (linear) way to increase transmitter power and/or receiver sensitivity[13]—is maximum when g reaches 0.42D. *After this, it profits the system less and less to increase range further.* We have a classic case of diminishing marginal returns.

Figure A3.5 How cumulative C_{pot} varies with communication range, g ($M \gg 0$).

From these basic considerations, our theory thus predicts that all highly evolved systems, from firms to living cells, will be found to have (1) high levels of connectivity up to 0.42D, (2) maximum connectivity at 0.42D, and (3) far less than full-possible-connectivity at scales, say, of 2D/3 and larger. Our theory also predicts that when cumulative C_{pot} gains per unit increase in range diminish to some threshold level, the system will turn instead to growing in absolute size (i.e. increase in diameter) or node density (i.e. increase in N for a given D). Failure to pull either of these game-plans off, however, might cause the system to break itself down into smaller subsystems each of which has the characteristics described in the previous sentence, and then to grow through loose agglomeration of more and more of these smaller-sized systems, each in themselves closer to optimal with respect to range and Ω.

The brain seems to have followed the latter strategy. For all its magnificence, the human brain's 30 billion neurons are nowhere *near* as connected as they theoretically could be in the volume of the skull: each neuron, on average, is connected to "only" a thousand others. And so we should not be surprised to find that "areas" of the brain are specialized in function and that these areas work with relatively limited communication to other areas.[14] Liike committees in some organization—these areas send only their resolutions to each other, not their deliberations. Although in in microprocessor design, increasing node (i.e. transistor) density is still an option, connecting them is still a space consuming process, meaning that chips too will continue to have "areas" that do different things internally than they communicate externally.

How might we represent all this in the "space" of complexity, C, and organization, R, that is, in Figures such as 2.1 of Chapter Two?

Well, returning the number of scales to four ($M = 4$) for illustrative purposes, we simply place not *one* but a constellation of *four* dots on the Ω-surface for each system we wish to represent. Each dot's placement tells us, simultaneously: the degree of actual complexity, C, the degree of potential complexity, C_{pot}, the degree of organization, R, and the level of Ω in the system *at that certain scale.* Then we can then join the dots with lines to indicate they belong together in a single, functional, spatially coherent system.[15] Thus:

Figure A3.6 Representing Ω at multiple scales in a single hypothetical system on the Ω-surface; one of indefinitely many possible 4-dot patterns describing various systems with $M = 4$.

Note that the scale denoted "medium" is likely to be farthest away from the origin, the scale denoted "medium-large" or "-long" is likely the second farthest out, the scale denote "small" or "short" next, and "large" or "long" scale closest in. This reflects the underlying alpha-distribution of potential complexity. (Recall that distance-from-the-origin (0, 0) is a measure of C_{pot}, as described in Chapter Two). Every scale, from macro to micro, can embody a certain range of values of Ω, of C and of R, from zero to some maximum; but each is constrained forever to its C_{pot} "orbit" around the origin. There are only two cures for this limitation, as I mentioned earlier: to grow in absolute size or to grow in node density. Both increase N and therefore C_{pot}. (Fractals patterns are interesting in this regard: with appropriate scale renormalization they may be represented by a single dot on the Ω-surface.)

In Chapter Two (pp... – ...) I tried to convey, with appropriate awe, how much of the world's complexity is hidden from us. Not just because the interiors of things are often shrouded by skins and shells, and not just because our unaided eyes cannot see very small things or detect the entire electromagnetic spectrum, etc.—these are physical limitations—but by the fact that we have *cognitive* limitations. We are only so smart. We see much of the world as though as in a cave—Plato's cave, to be exact, but where the shadows cast are not just our own shadows from a flickering fire and on a irregular wall (we are not so completely deluded!), but also from the outdoors, through an open cave mouth out of which we can see a certain amount directly. What appears in this, our "cave of consciousness," must pass two tests: (1) the physical medium through which information from the object/system is transmitted to us must be one to which we have the requisite physiological sensitivity, and (2) the complexity-and-organization of the object/system must lie within our capacity for making sense of it.

To help us with the first test we have invented scientific instruments, such as telescopes, microscopes, radiation detectors, tape recorders, "atom smashers," and the like. These allow us to see reality farther away, at larger scales, and closer by, at smaller ones. The second, cognitive test is harder to do well at, as it were: to push the metaphor perhaps a bit too far, they involve moving closer to the mouth of the cave, or widening it. The point is: we experience only those things and those "aspects" of things whose complexity-and-organization *at some scale or another* fall into our cognitive range, into our cave, which is "located" in the lower left-hand corner of the Ω-surface-entire. Such knowledge as we do have of the multitude of sub- and super-systems operating at scales and degrees of complexity that lie farther away is just that: "knowledge *of*," knowledge *about*. It is the sort of knowledge we code into diagrams, lists, equations, theories, inferences, pictures, maps, abstractions, computer simulations, and intuitions that are sometimes, to be sure, overwhelming to us in their own complexity, but each of which is a considerable (if still useful) reduction of the degree of complexity and/or degree of organization of the actual thing at *its* own various scales of operation. The triumph of science—as of art, as of philosophical reflection—is to help us become aware of the universe beyond immediate appearances and beyond easy comprehension, leaving pointers in the right place and right direction.[16]

Figure A3.7 tries to represent the situation in which we find ourselves. (Indeed, perforce, it *exemplifies* it.) Figure A3.7 shows that those scales of systems whose degree of complexity or organization lie outside our "cave of consciousness" are seen perspectivally from the cave in the correct direction. The correctness of the direction derives from the soundness of our estimate of the ratio of C to R, knowing neither number with any great accuracy. Of course, countless system "dots"—degrees of complexity and or organization at various scales—lying outside the cave share the same ratio of C to R, and are thus apt to be seen superposed—equivalent—in complexity-and-organization when they really are not. (To add an analogy to an analogy: what we see of the stars is what pierces the vault of our local Ω sky: we arrange them in constellations which ignore their actual remoteness.)

Figure A3.7 Two systems, each represented at four scales, showing scales outside the cave of consciousness projected back in.

What we have learned so far? That the appropriate response to the question "How complex is A?" is at least two further questions: "In what regard?" and "At what scale?" Sometimes, though, we might want to summarize. Acknowledging the risk, we might want to represent the average or total Ω of a system. How are we credibly to do this while knowing that the real complexity-and-organization of a system is being "performed" at scales all over the Ω-surface? It's not quite like adding up (an averaging) dollars or inches or degrees of temperature.

The problem is analogous to the problem of talking simultaneously about the *average altitude* of a team of climbers on some mountain *and* their *average position.* Each member of the team is a dot on the Ω-surface representing the degree of complexity and the degree of organization of the system at a particular scale and in some certain regard.[17] The aim of each climber is to move to the ridge of Ω along the arc or "orbit" of their own C_{pot} as fast as possible and thus to contribute maximally to the team's average (and total) altitude or Ω. On the assumption that there is little or no informational redundancy between the scales—i.e. that each scale, unlike a fractal pattern, is telling a different story—the way to figure their average altitude on the Ω-surface is unproblematic: simply add the altitudes of every one of the M climbers together and divide the total by M. Presto: Ω_{avg}. The climbers' average horizontal *position* on the surface, is almost as easily reckoned: add their C values and divide by M giving C_{avg}; then add their R values and divide by M giving R_{avg}; then locate a dot with coordinates C_{avg} and R_{avg} on the surface. This dot, this virtual climber, will be at the "center of mass" or centroid of the M climbers representing the system. Call this position: X.

Now comes the problem: it is extremely unlikely that X will lie anywhere on the contour of Ω_{avg}. Far more likely is it that Ω_{avg} will lie directly *under* the Ω-surface at X, and this is graphically misleading

The remedy is partial; a sacrifice must be made: extend a line from the origin to X, and where it crosses the contour of Ω_{avg} place the revised dot, X'. Viewed from the origin, as it were, X' retains the important bearing information R_{avg}/C_{avg} while giving an accurate reading of Ω_{avg}. Sacrificed, of course, is the correct, viewed-from-above, average position of all the climbers. All of this is shown in Figure A3.8.[18]

Figure A3.8 Representing the average magnitude of Ω appropriately on the Ω-surface.

In Chapters Three and Four I begin to link *total satisfaction,* $\boldsymbol{S}$, to Ω, and happiness, $\Delta\boldsymbol{S}$, to value, $\Delta\Omega$. I posit six needs and a satisfaction level, S_i (i = 1,2...6), associated with each need such that $\boldsymbol{S}$ is not the simple summation of those satisfaction levels, as one might expect, but rather $\boldsymbol{S} = S_1(1 + S_2(1 + S_3(1 + S_4(1 + S_5(1 + S_6)))))$. The connection between need-satisfaction-level and Ω-at-a-given-scale-in-a-system is not a strong one. Satisfying the need, say, for legitimacy, does not clearly involve a particular *scale* of social interaction (or neural function), a scale that is different from the one involved in satisfying the need, say, for confidence. Lower needs and higher needs do not map onto corresponding small and large scales as cleanly as it does in music, where *timbre* maps to small scale and *melody* maps to larger scales of acoustic patterning, and so on. Rather, individual needs and their satisfaction operate with and across dozens of scales of interaction and information simultaneously. To push the analogy, each need is like a whole *genre* of music. Such is their total complexity.

We should note again that the phase space created by putting R on the Y-axis and C on the X-axis of a graph is not a smooth and continuous space. Nor, therefore, is the Ω-surface a smooth and continuous one. In the formalism of Shannon-Weaver information theory upon which ours is based, there are always a discrete and finite number of states that a system could be in. That is, N is always a positive integer less than infinity. This means that C_{pot}, which is equal to $\log_2 N$, cannot take on any numerical value *between*, for example, 3 (for $N = 8$) and 3.17 (for $N = 9$), or between 4.25 (for $N = 19$) and 4.32 (for $N = 20$). Information is a quantum-like phenomenon.

Another way to say this: there are combinations of C and R that are "impossible" because $C^2 + R^2$ would not equal log(base 2) of *any* integer. (For example, $C = 2.4$, $R = 2.6$).

What we *can* say, though, is that the Ω-surface becomes smoother and more continuous—i.e. more like a "real" surface—as N gets larger and the C_{pot} contours get closer and closer together. If it is to evolve outward and away from the origin, a system must "jump" from one ring or arc of C_{pot} to the next the next higher one, not unlike electrons around an atomic nucleus. One might call this "the Bohr-Pauli rule," in analogy—and deference—to Niels Bohr's

1913 planetary model of the atom, and Wolfgang Pauli's 1925 discovery that no two electrons can share the same orbit. Whether the analogy is trivial or profound, I shall leave for someone better qualified to say.

The Bohr-Pauli rule yields further insight into some of the problems and opportunities of dealing with systems that carry different amount of Ω at different scales.

Take a symbol string made up of an "alphabet" of four different symbols. Let the string be N symbols in total length. At the scale of a single symbol (i.e. taking one at a time) C_{pot} = log(4) = 2 bits per symbol. The maximum possible amount of information the whole string can convey, in and of itself, is thus $2N$ bits. (I say "in and of itself" because a rather modest-size string—indeed, a one bit "string"—can *trigger* gigantic cascades of information from a larger system.) If we take the same string *two* symbols at a time (i.e. in *pairs*), then C_{pot} = 4 bits/pair. The maximum information conveyable by the whole string is still $2N$ bits, because, although each pair of symbols can carry twice as much information, *there are half as many pairs in the whole string.* Continue this analysis for 3 symbols at a time, 4, 5...until you get to N of which there is only one example. (This is an analysis by "group length," but it could also be by wavelength [using Fourier analysis], number of components in an object, etc.).

Now, measure the frequency distribution of actual instances of possible groupings for each group length. The relative frequency of each logically possible group (in each group-length category) gives us a probability distribution. This probability distribution would guide our expectations as to what a new symbol or symbol-sequence would "have" to be, given what we had and what was recently added to it. Method: Let M be the length of the group under consideration. Measure C_M With C_M in hand and with $C_{pot,\,M} = 2M$, calculate R and put a dot in the appropriate place on the graph of C and R. What we will find is that *all dots lie to the right and "up" of any dot made by a smaller group length..* This is because the complexity and/or organization of a sequence of symbols cannot be lower than the complexity or organization of any sequence that it subsumes. Call this the "subsumption rule."

This is all better illustrated than verbalized:

Figure A 3.9 An array of dots on the Ω-surface (contours not shown) representing increasing magnitudes of $C_{pot} = \log_2 2M$ (M being the group-length analyzed) of a single linear string of N symbols. The gray dots are impossible because of the Bohr-Pauli rule. The black dots are impossible because of the subsumption rule.[19]

At scales spatiotemporally larger than the system as a whole, *the subsumption rule ceases to apply* and a dot can once again exist anywhere along a C_{pot} arc. For example, it makes sense to ask whether or not Charles is in the woodshed and to put $N = 2$. (I.e., possiblity 1: *he is*. Possibility 2: *he isn't.*) The full physical reality of Charles's being in the wood shed or not however, is vast in its complexity. Charles breathes, Charles touches things; he radiates heat, he thinks. He is a universe unto himself. His presence/absence is rich in consequentiality for the the things and creatures in the woodshed, not to mention for Charles himself. All this we overlook entirely when we want to know whether Charles *as a whole* is, or is not, in woodshed, and then, not knowing for sure, estimate the probability of each. Similarly, when we listen for *melody*, we ignore the filigreed microstructure that makes up each note's timbre. This is how we treat most of things in the world around us: we see, notice, and ask questions that are answerable from within the cave of consciousness—limited, as we learned in Chapter Four, to dealing with a handful of possibilites at any one time. I represent this very schematically in Figure A 3.10.

Figure A 3.10 The black dot represents a scale larger than the whole, which thus can break the subsumption rule. It too is projected back into the "cave of consciousness" (shaded area).

Finally this, a concrete suggestion! My hypothesis is that living systems lie at or near the ridge of Ω at *all* scales, although not all of those scales of operation would be readily perceptible to us. In the search for extra-terrestrial intelligence from interstellar radio signals (the SETI project), I would suggest testing for optimal-Ω across all temporal group-lengths and signal wavelengths in the discretized signal. Complete regularity (i.e., $C = 0$, $R > 0$) is no more a sign of life than complete randomness would be (i.e., $C > 0$, $R = 0$). But $C = R > 0$ *could* be the sign of life we are looking for .

NOTES to Appendix 3: "On Omega at Different Scales"

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[1] See Stuart Kaufman's chapters on "random graphs" in his *Origins of Order* (New York, Oxford University Press, 1993) pp. 198 ff.

Let K be the average number of links per node. That is, let $K = L_{\text{total}}/N$. When N is large and $L_{\text{total}} = 2N - 1$, $K \approx 2$. Kaufman discusses such "K = 2" graphs at length (he calls them "random Boolean networks"), and cites research that shows that K = 2 represents a critical level of connectivity: the threshold at which islands of subsystems begin to appear in cellular automata, a level of internal freedom-of-parts that give the system as a whole some "life," but before it breaks apart completely with internal disconnectedness.

In graph theory, a graph is said to be "connected"if there is a *path* (consisting of one of several successive links) between every pair of nodes. For simplicity, I do not use this concept in this text.

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[2] True and interesting: in cities, the correlation between crow-fly and actual distances increases with the average distance between nodes sampled.

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[3] Actually, the number of scales, M, has to be finite. Indeed, to be a useful analytical tool, M should be far smaller than L_{total}

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[4] We can define the diameter of a system generally as the straight-line distance between the two farthest-apart nodes.

[5] More generally stated, $L_{total} = \sum_{m=1}^{M} L_m \leq N(N-1)/2$, ($m$ = 1, 2, 3...M) and L is the number of links at scale m and no link can belong to two scales.

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[6] In a *lattice*, which is one kind of graph, all nodes are connected to other nodes (but not all other nodes) at some particular scale. Think of a square grid of links with nodes at every intersection.

Duncan J. Watts and Steven H. Strogatz "Collective dynamics of 'small-world' networks," *Nature*, Vol 393, June 1998, pp. 440-442, show that lattices very rapidly become better connected (in terms of having lower average number of intervening nodes between any two randomly chosen nodes) as only a few random long-range links are made. Watts and Strogatz call such graphs "small-world networks." "The neural network of the worm *Caenorhabditis Elegans*, the power grid of the western United States, and the collaborative graph of film actors are all shown to be small-world networks." So are typical city streets linked by train or subway stops, they might have added. In structure, small-world networks lie "somewhere between completely regular and completely random graphs," say Watts and Strogatz. Surprise.

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[7] The average distance between two points on a line segment of length d is $d/3$. (Calculation courtesy of Prof. Nikos Salingaros, University of Texas at San Antonio, 1997.) For straight lines inside of a three dimensional sphere of diameter D it is approximately $D/3$ also. The fact that for a circular disc the average length (correct to three places) is $0.453D$, the median length is $0.447D$, and *the most probable* is $0.418D$ (which is itself a number very, very close to halfway between one-third and one-half the diameter), is interesting and surprising. See F. Garwood and J. C. Tanner, "On Note 2754—a repeated integral" *The Mathematical Gazette*, Vol XLII #342, 1958, pp. 292-293, and David Fairthorne, "The distances between random points in two concentric circles" *Biometrica SI*, 1964. pp. 275-277. Thanks also to Dr. Ricardo Garcia Pelayo at the University of Madrid (personal communcation, 1997).

To work with a computer simulation that confirms and illustrates some of these results, visit http://mather.ar.utexas.edu/cadlab/turknett/javacode/experiment1/Experiment1.html (for the circular disc) and http://mather.ar.utexas.edu/cadlab/turknett/javacode/experiment2/Experiment2.html (the sphere), written in Java by Robert Turknett of the University of Texas at Austin, 1997.

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[8] If for no other reason than that the interval 0-to-∞ cannot be divided into a finite number of equal intervals.

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[9] On this basis, the value of R for the interior of a circle (i.e. a disc) sorted into four scales in 0.76 bits. Sorted into 100 scales it is close to 2 bits. For other shapes it would be different (higher) given decomposition into the same number of scales.

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[10] The inverse square law says that the effect of A, emitting energy, on B situated a distance $d_{A,B}$ apart is proportional to $1/(d_{A,B})^2$. Actually, any monotonic function relating signal-effect-strength to distance will suffice to parse out our scales using receiver sensitivity.

In most biological and social systems in fact, parts of the system communicate with each other along specific channels and media that have specific ranges of effectiveness. These may or may not "religiously" obey the inverse square law: they may become clogged or blocked or noisy with distance, they may be effectively immune to the inverse square law over large distances (as signals in laser beams and fibre-optic cable are) but easily be cut off, blocked, and so on. No matter: actual connectivity will be a subset of full connectivity, and the system will generally be parse-able into node groupings having similar-length links. Even on the internet, which would seem to transcend distance effects entirely due to the use of optical cables, satellites, and digital packet switching and error correcting techniques, there is a strong correlation between achievable communication rates (measured in bits/sec) and the physical distance from one computer to another.

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[11] Nothing mysterious is being suggested here. The scale analysis could set one of the M scales to be so fine as to capture the internal workings of nodes. After all, what is a "node" but a subsystem of scale much smaller than the smallest one under present study?

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[12] As similar as they are, the intended relationship between the variables "l" and "g" is as follows: l refers to the geodesic or Euclidean distance between two nodes, g refers to the effective communication range of a typical node to others. That is, g is thought of as a radius, and thus calls out links to all nodes of length (distance) $l \leq g$ around the transmitting node.

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[13] In the case of transmitter power, signal strength at a given distance away from the transmitter is a linearly proportional to transmitter power (read energy consumption). For receiver sensitivity, the relationship to energy consumption is less clear (although, at least in radio technology, it does take extra energy to boost weak signals once detected and more material—in antennas and such—to detect them in the first place).

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[14] William H. Calvin in *The Cerebral Code* (Cambridge, Mass., MIT Press, 1996) p. 34–35, finds the effective radius of excitation around the average human neuron to be around 0.5mm. This range limit creates a gigantic triangular-hexagonal latticework of interconnected neurons space mutually 0.5mm apart which may or may not become coordinated at higher (larger) scales yet.

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[15] Note that these "links" do not *necessarily* represent communication "between scales" as they did between nodes, although in a more patient analysis this idea might prove fruitful. For starters, is our road system not already stratified into interstate freeways, highways, farm-to-market toads, local roads, neighborhood streets, driveways...each with their typical ever-shorter length/distance between intersections? And when one drives from one's house to Grandmother's house across the country for Thanksgiving, does one not step from scale to scale, driveway up to freeway and back down to driveway again though all the other scales?

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[16] The reader may find the idea that there is a definite limit to the actual complexity, C, we can comprehend more plausible than the idea that there is a limit to the degree of organization, R, we can comprehend. An extremely large checkerboard, which is simple and highly organized, seems to offer no particular challenge. On reflection, however, one reconsiders. An appreciation of the degree of something's organization, R, depends on an appreciation of how complex it *could*-be-but-isn't, i.e. an appreciation of C_{pot}. (Recall that $R^2 = C_{pot}^2 - C^2$.) I would suggest that "actual" potential complexity, and therefore actual degree of organization, is as easy to underestimate, "shortchange," and otherwise fail to appreciate as is actual complexity proper, i.e. C. I hinted at this with my thought experiment involving Paul and Quentin in Chapter One. When appreciating the symmetry of a flower, only a small fraction of the actual number of alternative permutations of petals, pistils, and stamens (that could still be "a flower") occur to us, let alone the vastly larger number of arrangements of these elements that would *not* be a

flower.

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[17] One might imagine an Olympic climbing competition with teams from different countries. Each team is a single system, and each member of a team represents a different scale in the system. We know they are a team because they are tied by a rope... With this, some intriguing analogies present themselves. For example, can one team-member, slipping, pull another down, or, gaining purchase, pull another up? Are they in some sense insurance for one another? Do members of the team communicate? (Translation: can one scale affect another, and if so how?) Etc. I suspect that only empirical work with different systems, from software packages to paramecia, can really pursue the analogies much further.

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[18] It is conceivable that the complexity and organization of all scales of a given system lie outside the window of comprehension, but that the average magnitude of them, and of Ω, lies inside of it. This can happen when R is very high and C is very low for some scales, and C is very high and R is very low for others, making Ω for each very low and thus Ω_{avg} low too. Indeed low enough to comprehend. This should not present us with a problem. As Figure A3.7 proposes, what lies outside the window of comprehension is actually first projected into it, appropriately located in direction. The average position and average value of Ω of any constellation or team of points thus inside the window—some actually there, some projected there—will always lie inside the window too.

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[19] Note that Figures A3.6, A3.7 and A3.8 seem "impossible" in that they appear to break the subsumption rule. They do not, however, since they describe a different kind of system (a two-dimensional network rather than a string of symbols) and the relevant unit of information is the presence/absence of a link between two nodes.